Correspondence

The Extended Theory of the Manley-Rowe's Energy Relations in Nonlinear Elements and Nonlinear Lossless Medium*

Energy relations for a lumped nonlinear reactance excited by two different fundamental frequencies were derived by J. M. Manley and H. E. Rowe [1]. R. H. Pantell [10] developed the energy relations in a nonlinear resistive element. Furthermore, H. A. Haus extended the Manley-Rowe's energy relations to a nonlinear lossless medium excited by two fundamental frequencies [2]. This paper extends the Manley-Rowe relations in the case of a lumped nonlinear element (lossless reactance and resistive element) and nonlinear lossless reciprocal medium excited by k numbers of fundamental frequencies, wherein the extended relations consist of k independent equations.

I. In the Case of Nonlinear Lossless Reactance

In this case, generalized energy relations have been independently derived by this author [6] in Japan, and Yeh in America [9].

The author derived the extended relations by using the multiple Fourier Series [6], [9].

The energy relations obtained in both papers have the same results and consist of k independent equations. This relation is also obtained by using the energy conservation theory directly [8]. This extended energy relation can be applied to the multi-pumping parametric amplifier. If there are $(k-1)$ pumping frequencies, $f_{p1}, f_{p2}, \cdots, f_{p(k-1)}$, and if we choose the idling frequency according to

$$f_i = \sum_{j=1}^{k-1} f_{pj} - f_s$$

(fs: signal frequency), the general energy relations in the multi-pumping parametric amplifier are derived as

$$\frac{W_s}{f_s} = \frac{W_i}{f_i},$$

$$\frac{W_{p1}}{f_{p1}} = \frac{W_{p2}}{f_{p2}} = \cdots = \frac{W_{p(k-1)}}{f_{p(k-1)}} = -\frac{W_i}{f_i}, \quad (1)$$

where

$$\sum_{j=1}^{k-1} f_{pj} > f_s.$$

II. In the Case of a Nonlinear Resistive Element

The energy relations in a nonlinear resistive element excited by two fundamental frequencies were derived by R. H. Pantel [10].

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We can extend this relation to the multi-exciting system by using multiple Fourier Series. Assume that,

1) The relations between the voltage v and the current i in a nonlinear resistor can be expressed by single valued function,

$$v = f(i): \text{single valued function}, \quad (2)$$

2) There are k numbers of fundamental frequencies $f_1, f_2, f_3, \cdots, f_k$ (k: positive integer). Higher harmonics and sidebands

$$\sum_{i=1}^{k} n_i f_i,$$

are generated by the nonlinearity.

Then, the voltage v and the current i may be expressed by the multiple Fourier Series as follows:

Furthermore, the coefficient of the Fourier Series in (3) can be expressed by (6).

$$V_{n_1 n_2 \cdots n_k} = \frac{1}{(2\pi)^k} \int_0^{2\pi} dx_k \int_0^{2\pi} dx_{k-1} \int_0^{2\pi} \cdots \cdot \int_0^{2\pi} dx_1 v e^{-j(n_1x_1+n_2x_2+\cdots+n_kx_k)}. \quad (6)$$

Let us define the average real power and the reactive power as follows:

$$W_{n_1n_2\cdots n_k} = V_{n_1n_2\cdots n_k} I^*_{n_1n_2\cdots n_k} + V^*_{n_1n_2\cdots n_k} I_{n_1n_2\cdots n_k}$$

$$X_{n_1n_2\cdots n_k} = -j[V_{n_1n_2\cdots n_k} I^*_{n_1n_2\cdots n_k} - V^*_{n_1n_2\cdots n_k} I_{n_1n_2\cdots n_k}]. \quad (7)$$

Finally, we can obtain the following k independent equations (8) and (9), by using (2)–(7) and by using the condition that v is the single valued function of i.

For the reactive power,

$$\left.\begin{array}{l}
\sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_2=-\infty}^{\infty} \sum_{n_1=0}^{\infty} n_1 X_{n_1n_2\cdots n_k} = 0 \\
\sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_2=0}^{\infty} \sum_{n_1=-\infty}^{\infty} n_2 X_{n_1n_2\cdots n_k} = 0 \\
\vdots \\
\sum_{n_k=0}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_2=-\infty}^{\infty} \sum_{n_1=-\infty}^{\infty} n_k X_{n_1n_2\cdots n_k} = 0
\end{array}\right\}. \quad (8)$$

For the real power,

$$\left.\begin{array}{l}
\sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_2=-\infty}^{\infty} \sum_{n_1=0}^{\infty} n_1^2 W_{n_1n_2\cdots n_k} = h_{n1} \\
\sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_2=0}^{\infty} \sum_{n_1=-\infty}^{\infty} n_2^2 W_{n_1n_2\cdots n_k} = h_{n2} \\
\vdots \\
\sum_{n_k=0}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_2=-\infty}^{\infty} \sum_{n_1=-\infty}^{\infty} n_k^2 W_{n_1n_2\cdots n_k} = h_{nk}
\end{array}\right\}, \quad (9)$$

$$v = \sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \cdot \sum_{n_1=-\infty}^{\infty} V_{n_1n_2\cdots n_k} e^{j(n_1x_1+n_2x_2+\cdots+n_kx_k)} \quad (3)$$

$$i = \sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \cdot \sum_{n_1=-\infty}^{\infty} I_{n_1n_2\cdots n_k} e^{j(n_1x_1+n_2x_2+\cdots+n_kx_k)} \quad (4)$$

where

$$x_i = \omega_i t. \qquad (i = 1, 2, \cdots k).$$

Since v and i are real quantities,

$$V_{n_1n_2\cdots n_k} = V^*_{-n_1-n_2\cdots -n_k},$$

$$V_{-n_1-n_2\cdots -n_k} = V^*_{n_1n_2\cdots n_k}$$

$$I_{n_1n_2\cdots n_k} = I^*_{-n_1-n_2\cdots -n_k},$$

$$I_{-n_1-n_2\cdots -n_k} = I^*_{n_1n_2\cdots n_k}. \quad (5)$$

where

$$h_{ni} = \frac{1}{(2\pi)^k} \int_0^{2\pi} dx_k \int_0^{2\pi} dx_{k-1} \cdots \cdot \int_0^{2\pi} dx_1 \int_0^{2\pi} dx_i \frac{\partial i}{\partial v} \left(\frac{\partial v}{\partial x_i}\right)^2.$$

III. Energy Relations in Nonlinear Lossless Medium With Multi-Excitations

Energy relations in nonlinear lossless medium have been published by H. A. Haus [2], whereby the number of excitations is confined to the case of two. The extended theory in nonlinear lossless medium which contains k excitations, and whose frequencies are $f_1, f_2, \cdots, f_k$, will be considered.

Assumed that,

1) the medium is nonlinear,
2) the medium is lossless, the conductivity $\sigma = 0$,
3) the medium is reciprocal, but is not necessary if required to be isotropic, and

4) there are no free charges, $\rho=0$,

Maxwell's equations are satisfied at any point in the medium

$$\nabla\times H = -\frac{\partial B}{\partial t} \quad \text{(a)}$$
$$\nabla\times E = \frac{\partial D}{\partial t} \quad \text{(b)}$$
$$B = \mu_0(H+M) \quad \text{(c)}$$
$$D = \epsilon_0 E + P \quad \text{(d)}$$
$$\text{div } D = 0, \quad \text{div } B = 0 \text{ (e)}. \qquad (10)$$

Let the medium be excited sinusoidally with respect to time, the excitation angular frequencies being ω_1, ω_2, ω_k. Their sideband and higher harmonics

$$\sum_{i=1}^{k} n_i\omega_i,$$

are generated by the nonlinearity.

At any point in the medium, the electric field E, whose frequency is $n_1\omega_1+n_2\omega_2+\cdots+n_k\omega_k$, can be expanded by the multiple Fourier Series

$$E(r) = \sum_{n_k=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty} E_{n_1n_2\cdots n_k}(r) \cdot e^{j(n_1\omega_1+n_2\omega_2+\cdots n_k\omega_k)t}, \qquad (11)$$

where r is the vector representing the position of observation from the origin.

Since E is real, it follows that

$$E_{n_1n_2\cdots n_k} = E^*_{n_1n_2\cdots n_k},$$
$$E_{-n_1-n_2\cdots-n_k} = E^*_{n_1n_2\cdots n_k}. \qquad (12)$$

These results are also valid for magnetic field H, electric polarization P, current density J, and magnetization M.

From (10a) and (10b),

$$\nabla\times E_{n_1n_2\cdots n_k} = -j\mu_0(n_1\omega_1+n_2\omega_2+\cdots+n_k\omega_k)\cdot(H_{n_1n_2\cdots n_k}+M_{n_1n_2\cdots n_k}) \qquad (13)$$

$$\nabla\times H_{n_1n_2\cdots n_k} = j(n_1\omega_1+n_2\omega_2+\cdots+n_k\omega_k)\cdot(\epsilon_0 E_{n_1n_2\cdots n_k}+P_{n_1n_2\cdots n_k}). \qquad (14)$$

Making the vector product $E_{n_1n_2\cdots n_k}\times H_{n_1n_2\cdots n_k}$ by (13) and (14) and applying the vector formula

$$\nabla\cdot(A\times B) = B\cdot\nabla\times A - A\cdot\nabla\times B;$$

and summing up with to the frequencies, from $-\infty$ to $+\infty$, k independent relations are obtained as follows,

$$\nabla\cdot\sum_{n_k=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty}\frac{n_iE_{n_1n_2\cdots n_k}\times H^*_{n_1n_2\cdots n_k}}{n_1\omega_1+n_2\omega_2+\cdots+n_k\omega_k}$$
$$= -j\sum_{n_k=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty} n_iM_{n_1\cdots n_k}\cdot H^*_{n_1n_2\cdots n_k}$$
$$+j\sum_{n_k=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty} n_iP^*_{n_1n_2\cdots n_k}\cdot E_{n_1n_2\cdots n_k}$$
$$i = 1, 2, \cdots, k. \qquad (15)$$

From assumption (2), $\sigma=0$, and (3), the functions describing the relations between H and M and between E and P are single valued, *i.e.*,

$$H = H(M),\ E = E(P) \cdots \text{Single Valued.} \qquad (16)$$

The energy supplied to the material of unit volume so as to cause magnetic polarization M, is given by the integral (16), H being a function of M,

$$\int_0^M H(M)dM. \qquad (17)$$

Since H is single valued, integral (17) is independent of the integrating path. The integral along the closed path vanishes, since the medium is lossless. The magnetic polarization M is expanded in the multiple Fourier Series at any point in the medium as,

$$M = \sum_{n_k=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty} M_{n_1n_2\cdots n_k}e^{j(n_1x_1+n_2x_2+\cdots n_kx_k)}$$
$$x_i = \omega_it \quad (i = 1, 2, \cdots k). \qquad (18)$$

The coefficient $M_{n_1n_2\cdots n_k}$ in (18) is expressed by,

$$M_{n_1n_2\cdots n_k} = \frac{1}{(2\pi)^k}\int_0^{2\pi}dx_k\int_0^{2\pi}dx_{k-1}\cdots\int_0^{2\pi}dx_iM(x_1x_2\cdots x_k)e^{-j(n_1x_1+n_2x_2+\cdots+n_kx_k)}$$
$$M_{n_1n_2\cdots n_k} = M^*_{-n_1-n_2\cdots-n_k},$$
$$M_{-n_1-n_2\cdots-n_k} = M^*_{n_1n_2\cdots n_k}, \qquad (19)$$

which may be rewritten as

$$H = H[M(x_1, x_2, \cdots x_k)] = H(x_1, x_2, \cdots, x_k). \qquad (20)$$

Likewise, H may be expressed by multiple Fourier Series.

By using the condition that $H(M)$ is single valued, the following equations are obtained.

$$\sum_{n_k=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty} jn_iH^*_{n_1n_2\cdots n_k}\cdot M_{n_1n_2\cdots n_k} = 0$$
$$(i = 1, 2, \cdots k). \qquad (21)$$

For P and E

$$\sum_{n_k=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty} jn_iP^*_{n_1n_2\cdots n_k}\cdot E_{n_1n_2\cdots n_k} = 0$$
$$(i = 1, 2, \cdots, k). \qquad (22)$$

Putting (21), (22) and $\sigma=0$, into (15), k independent equations are obtained in the lossless reciprocal medium excited by k fundamental frequencies. Thus

$$\nabla\cdot\sum_{n_k=-\infty}^{\infty}\sum_{n_{k-1}=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty}\frac{n_1E_{n_1n_2\cdots n_k}\times H^*_{n_1n_2\cdots n_k}}{n_1\omega_1+n_2\omega_2+\cdots+n_k\omega_k} = 0$$
$$\nabla\cdot\sum_{n_k=-\infty}^{\infty}\sum_{n_{k-1}=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty}\frac{n_2E_{n_1n_2\cdots n_k}\times H^*_{n_1n_2\cdots n_k}}{n_1\omega_1+n_2\omega_2+\cdots+n_k\omega_k} = 0 \qquad (23)$$
$$\vdots$$
$$\nabla\cdot\sum_{n_k=-\infty}^{\infty}\sum_{n_{k-1}=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty}\frac{n_kE_{n_1n_2\cdots n_k}\times H^*_{n_1n_2\cdots n_k}}{n_1\omega_1+n_2\omega_2+\cdots+n_k\omega_k} = 0.$$

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H. IWASAWA
Dept. of Wireless Engrg.
Okubo Works, Kobe Kogyo Corp.
Okubo, Akashi, Japan

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A Method of Improving Isolation in Multi-Channel Waveguide Systems*

In microwave measurements, frequent use is made of "two channel" or multiple channel systems, in order to obtain isolation from amplitude fluctuations in the generator output and/or various other benefits. In such systems it is generally required that the signal delivered to one channel be independent of changes in loading, etc., in the second channel, and a common problem is that of achieving or determining that an adequate degree of isolation exists between the different channels. It is the purpose of this letter to describe briefly procedures for obtaining and recognizing this condition.

Consider first the three arm junction of Fig. 1, where the generator feeds arm 1. The required condition for isolation is that

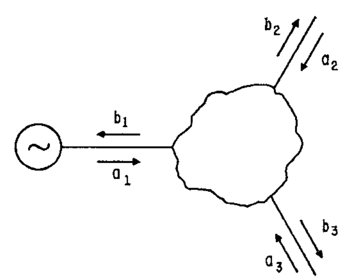

Fig. 1—Three arm waveguide junction

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